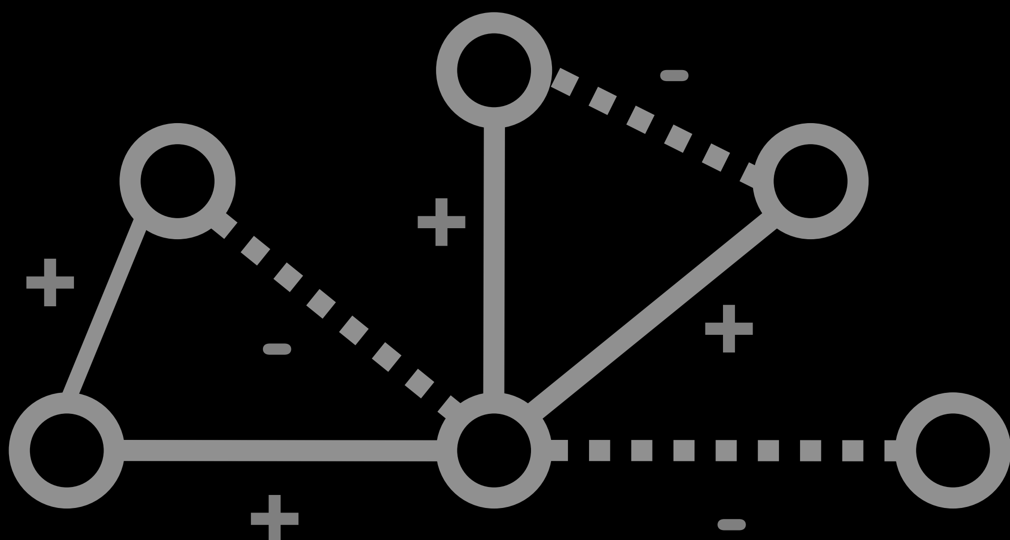
15-396

Science of teh Interwebs

Weak Ties, Triadic Closure and Structural Balance

Lecture 3 (September 6, 2011)

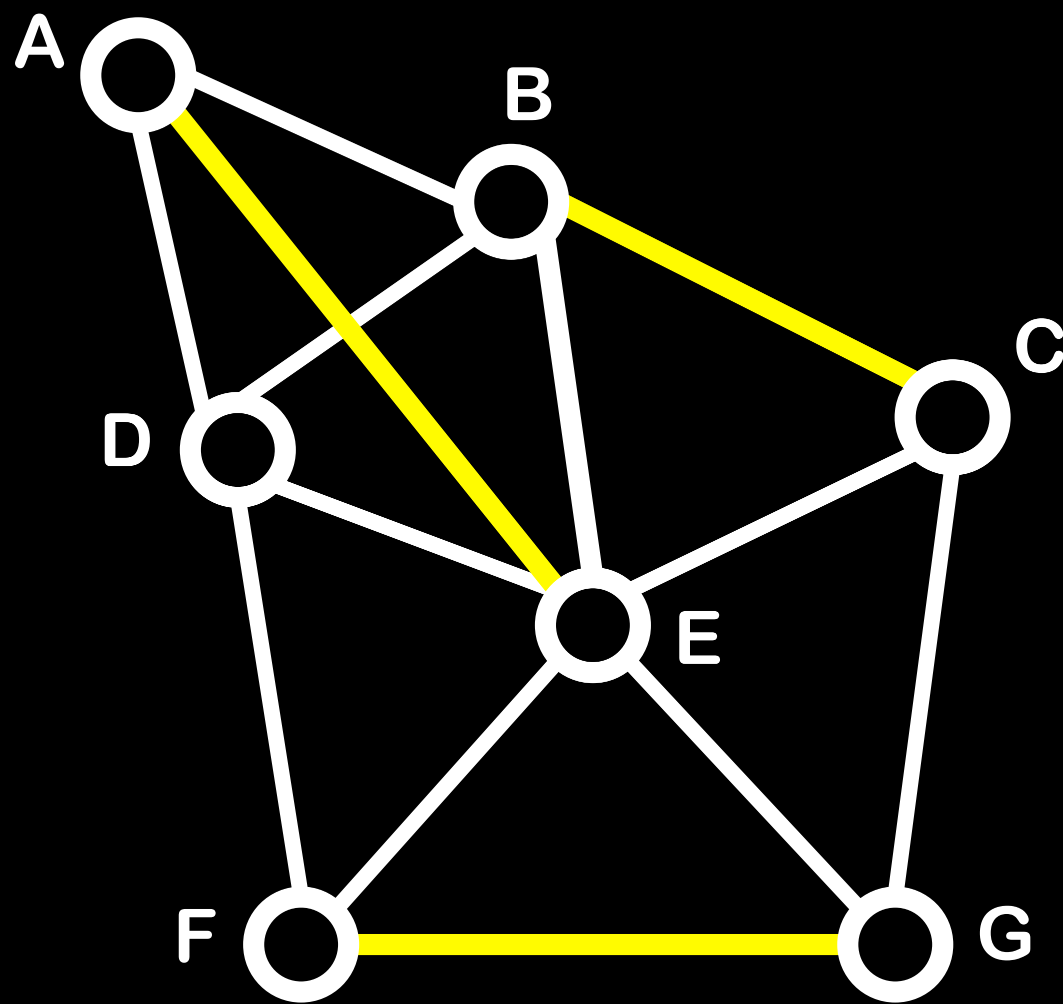


Homework 1 Information

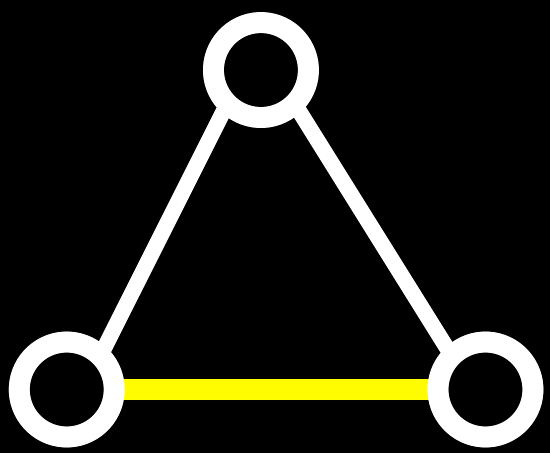
- Email your artifact to bmeeder@cs.cmu.edu with the subject "15-396 homework 1 artifact" whenever you have it ready.
- We have posted a 'cover sheet' that you **MUST** attach to your HW. Please fill it out so that we can read it. ☺
- **Failure to do so will result in 15 point penalty.**

Where do the best job leads come from: your close friends or your acquaintances?

How will this network evolve?

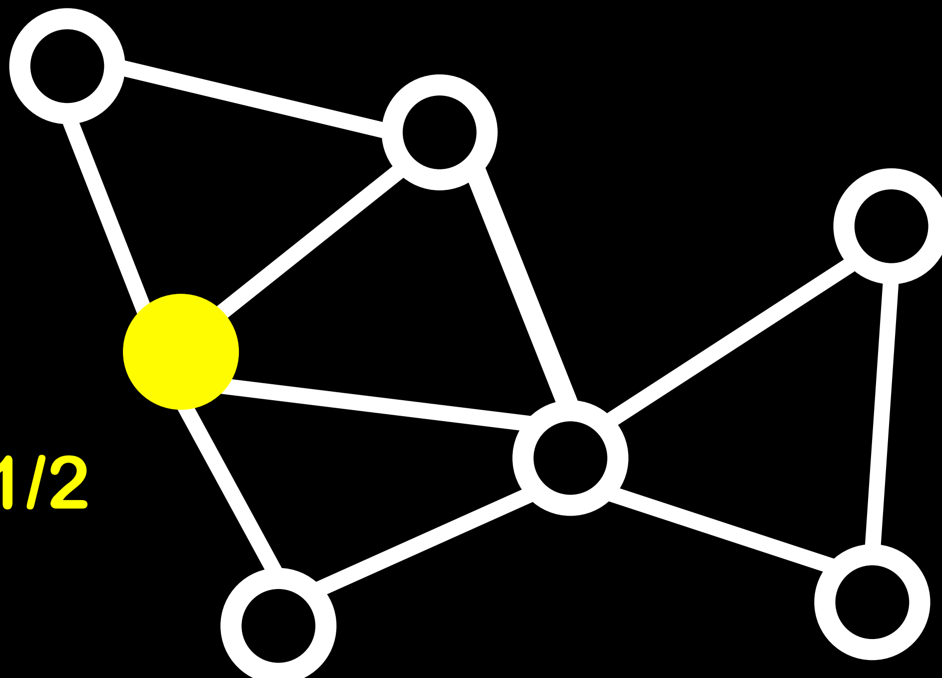


Triadic Closure: If two nodes have common neighbor, there is an increased likelihood that an edge between them forms



Definition: The clustering coefficient of a node v is the fraction of pairs of v's friends that are connected to each other by edges

Clustering Coefficient = 1/2



The higher the clustering coefficient of a node, the more strongly triadic closure is acting on it

Triangles in Graphs

The clustering coefficient is a local property.

Some global variants:

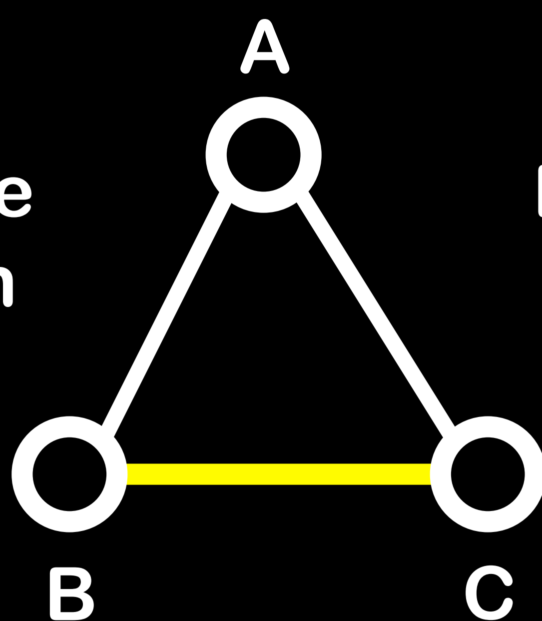
- Average clustering coefficient = Σ CC(v_i) / N
- Triangle density = # Triangles / (N choose 3)

Many real-world graphs exhibit greater clustering and higher triangle density than random graphs with similar properties.

Uses of Triadic Closure

- Friend suggestion on Facebook, Twitter, etc.
- Graph clustering
- Estimating tie strength between nodes
- Sociological theories of balance and exchange:

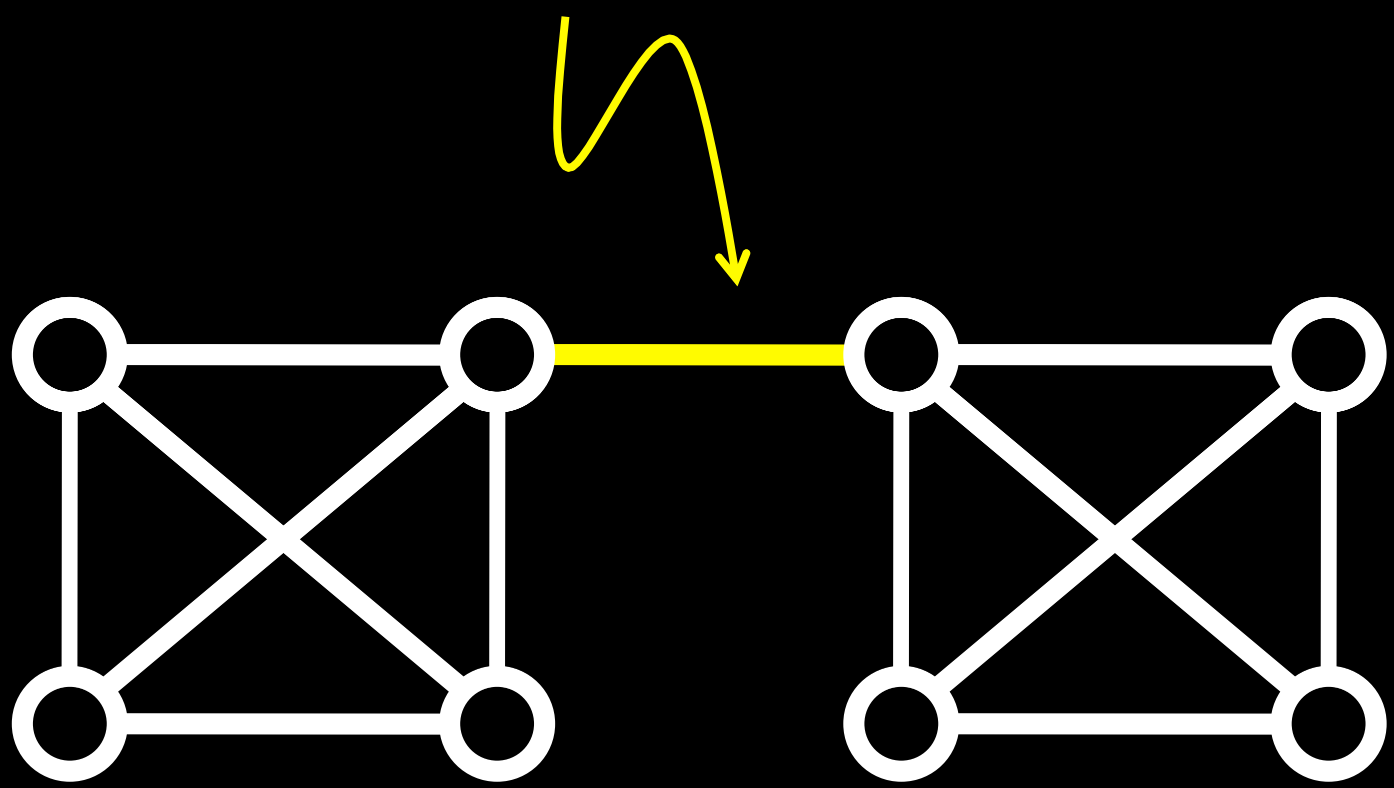
Balance: B-C tie can strengthen A-B tie



Exchange: B-C tie can weaken A-B tie

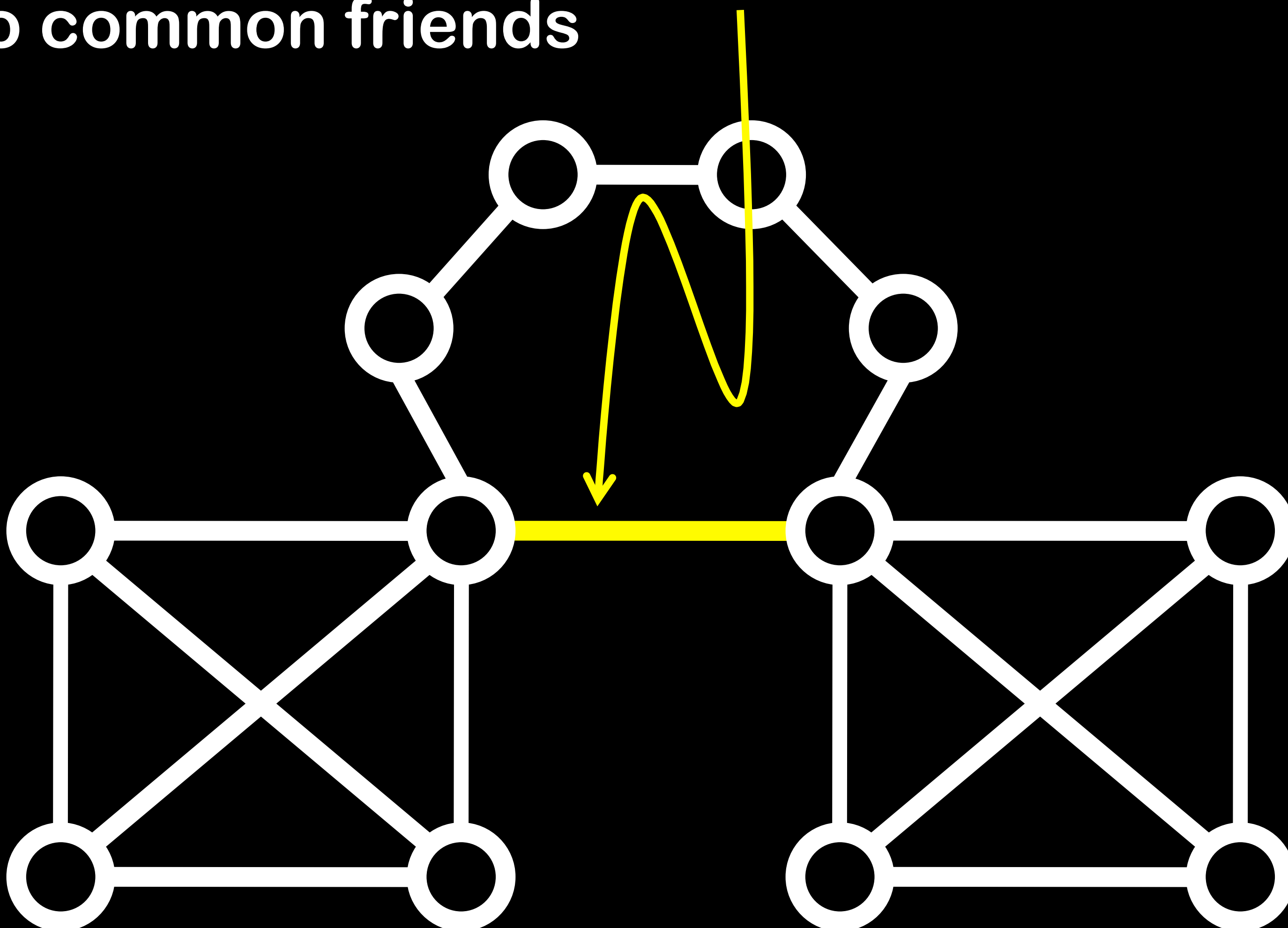
Bridges

An edge is a **bridge** if deleting it would cause its endpoints to lie in different components

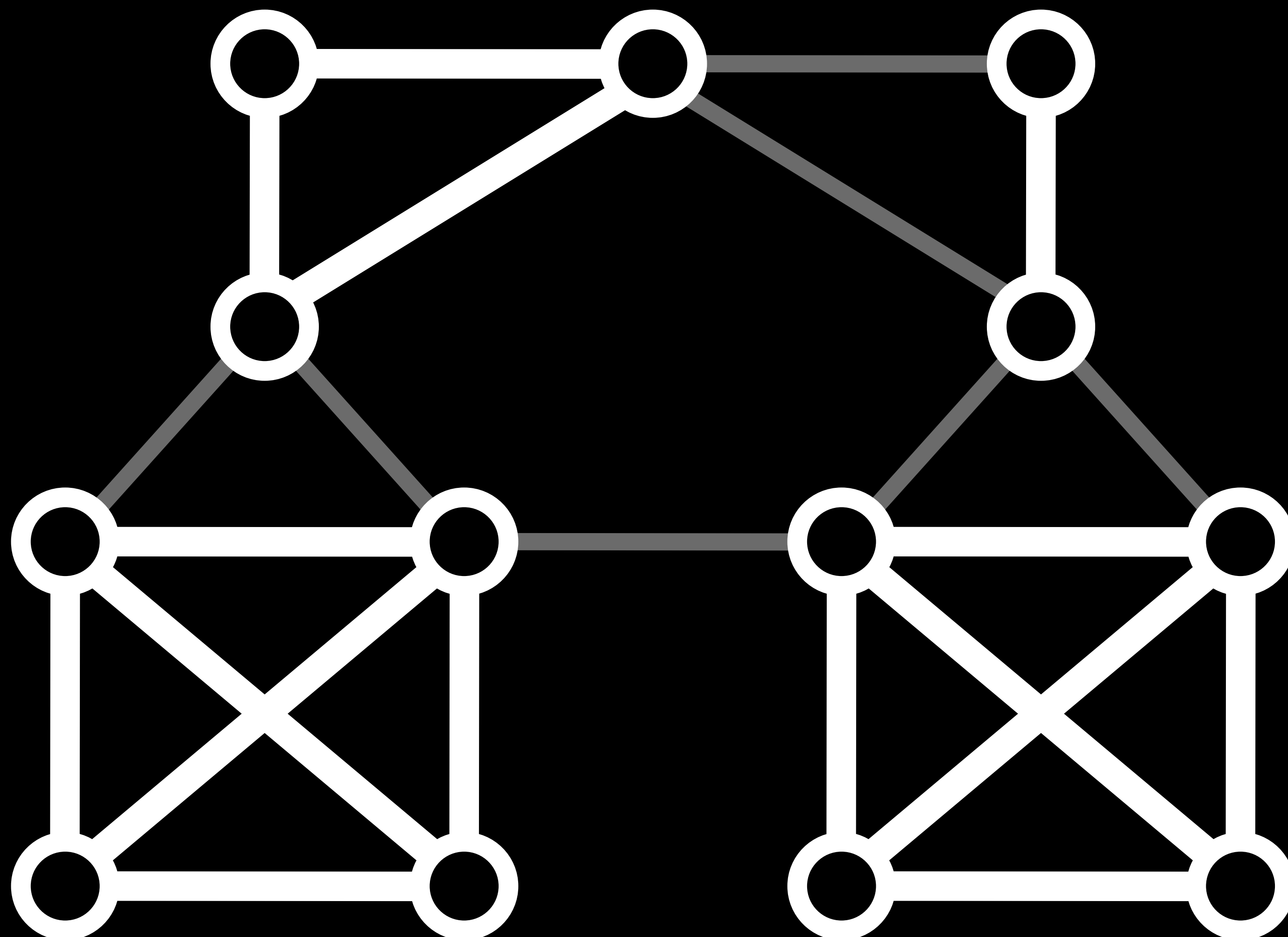


Local Bridges

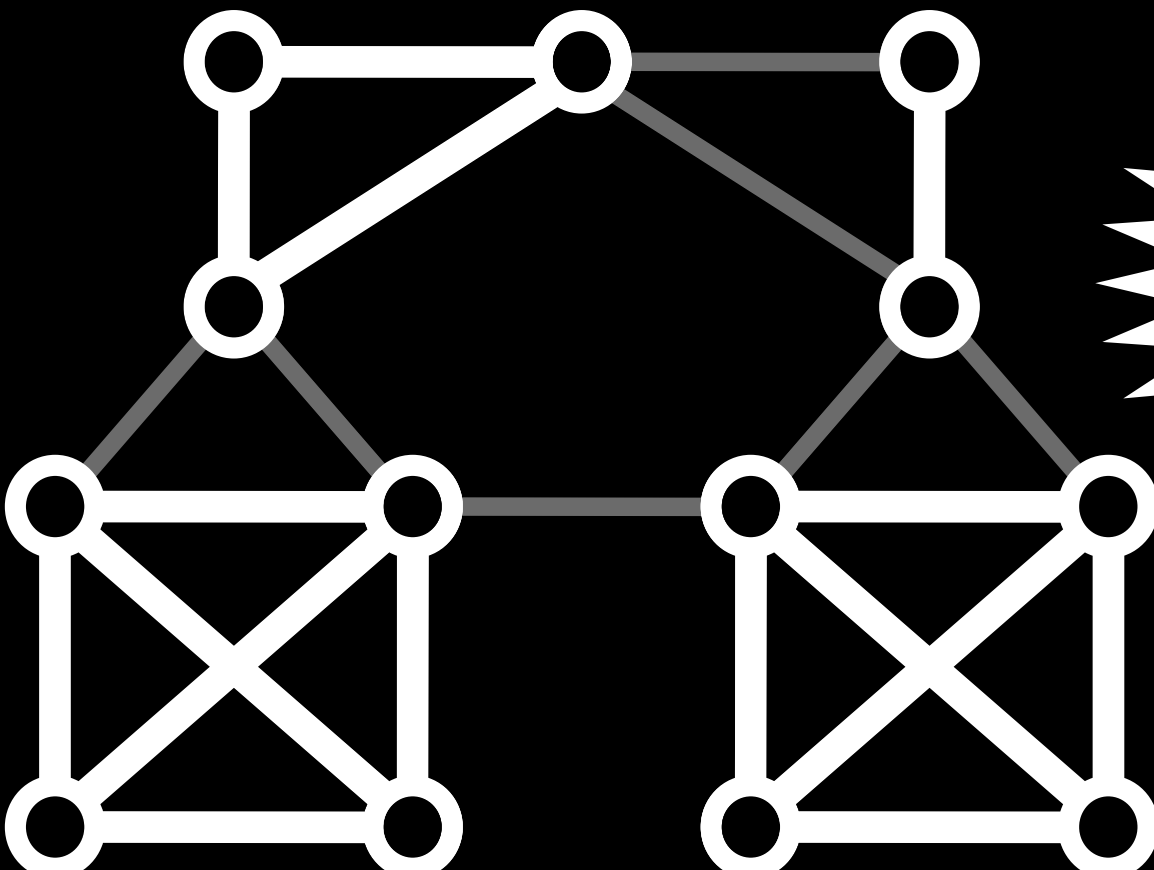
An edge is a **local bridge** if its endpoints have no common friends

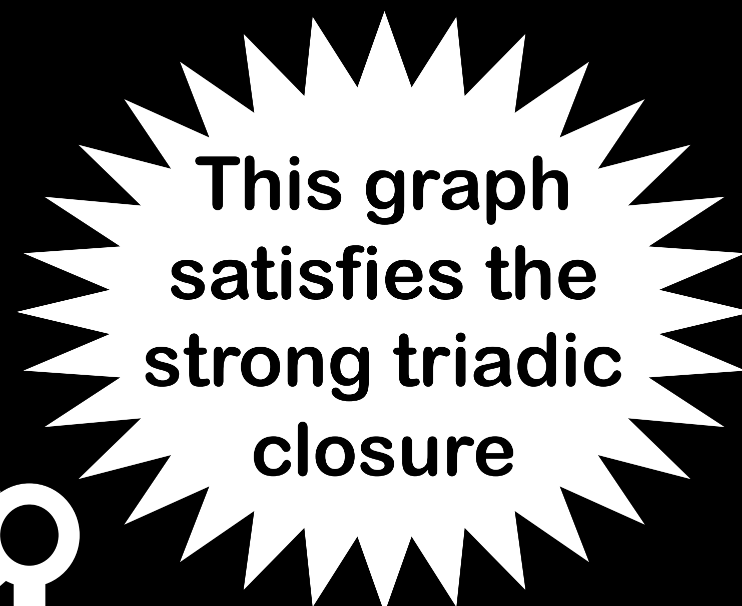


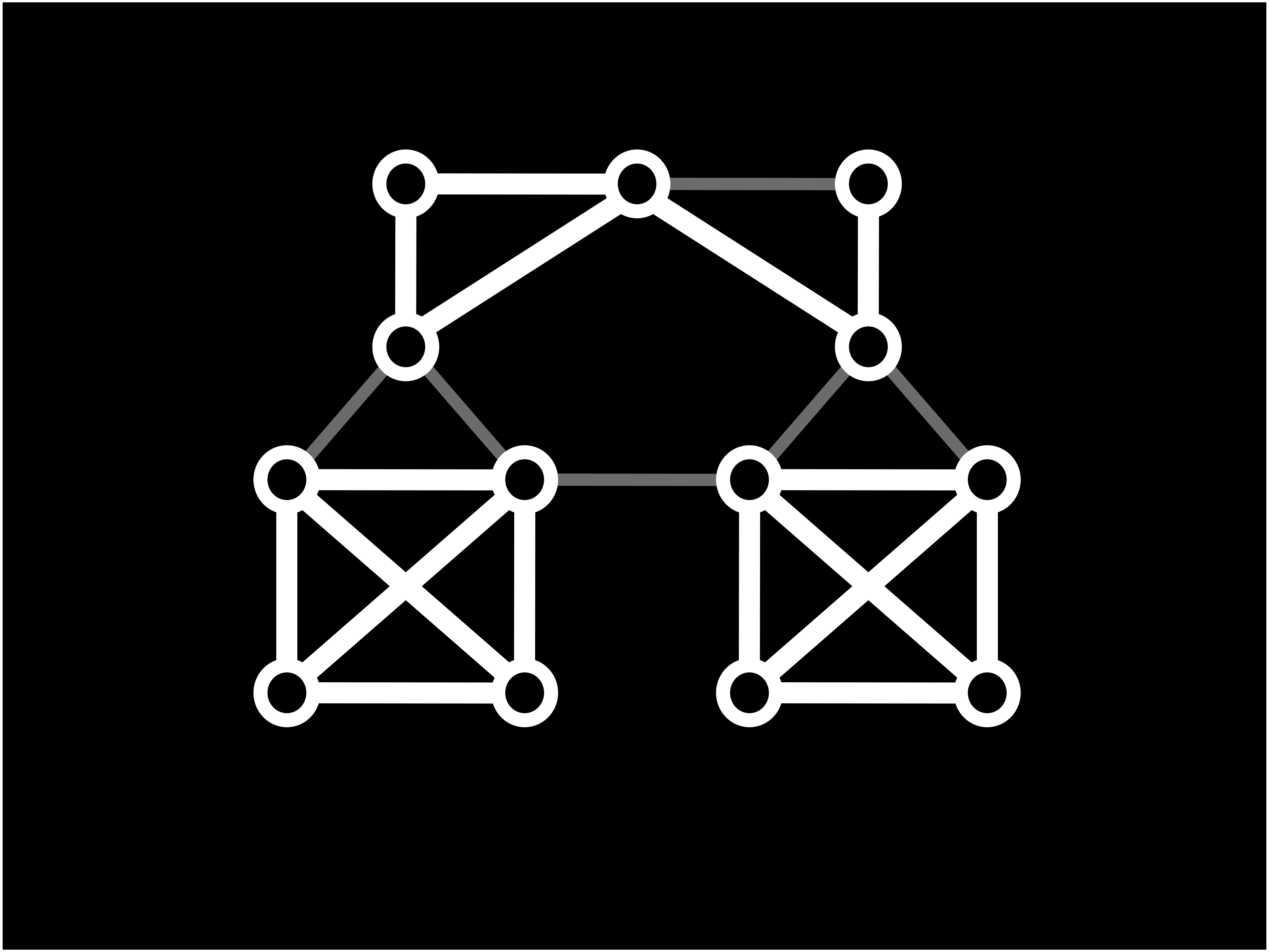
Weak Versus Strong Ties



Definition: Node v satisfies the **Strong Triadic Closure** if, for any two nodes u and w to which it has strong ties, there is an edge between u and w (which can be either weak or strong)



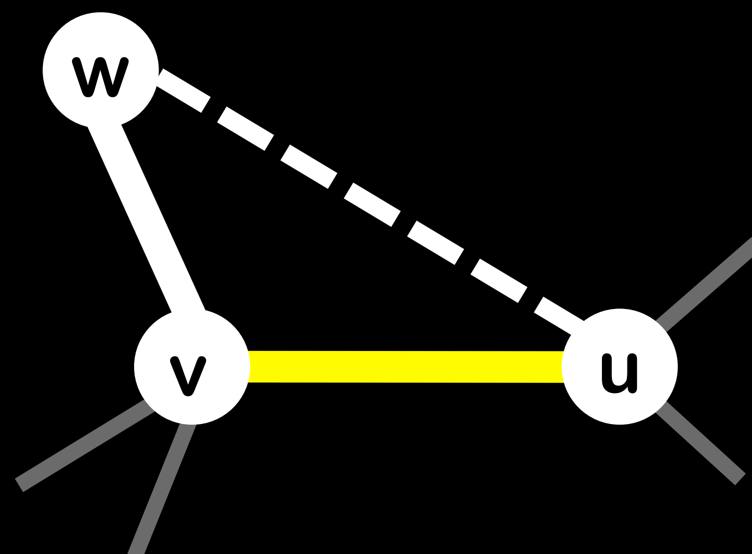




Theorem: If node v satisfies the Strong Triadic Closure and is involved in at least two strong ties, then any local bridge it is involved in must be a weak tie

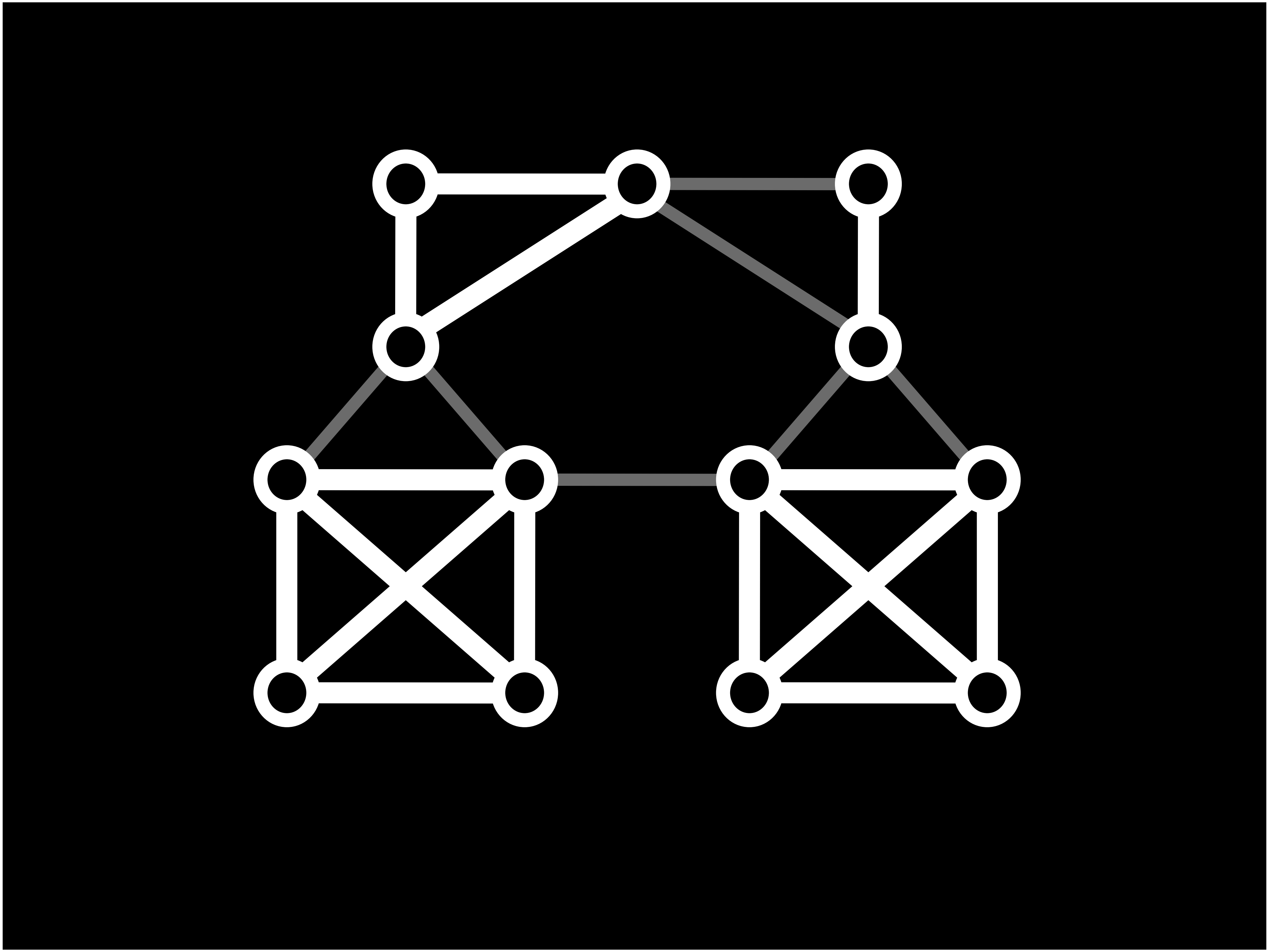
Proof (by contradiction):

Suppose edge v-u is a local bridge and it is a strong tie

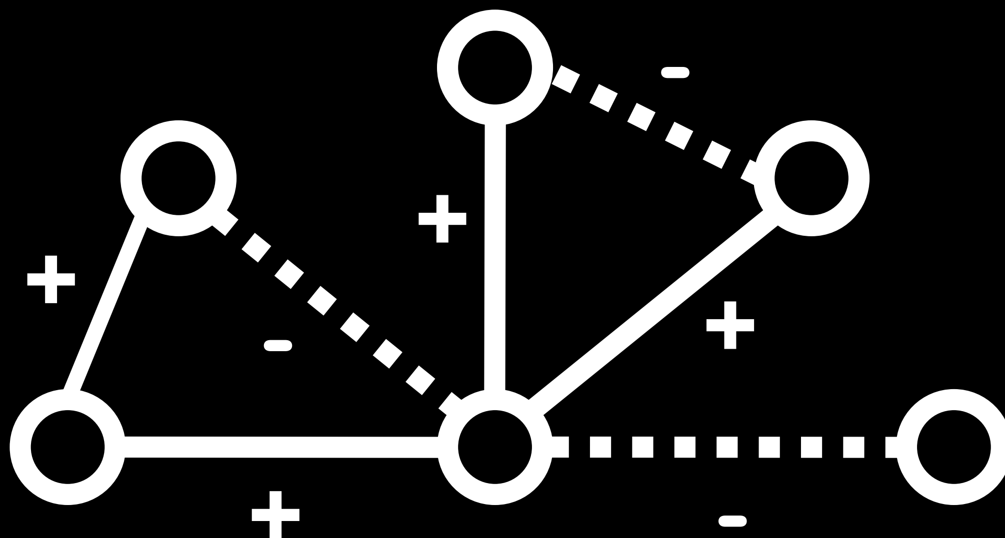


Then u-w must exist because of Strong Triadic Closure

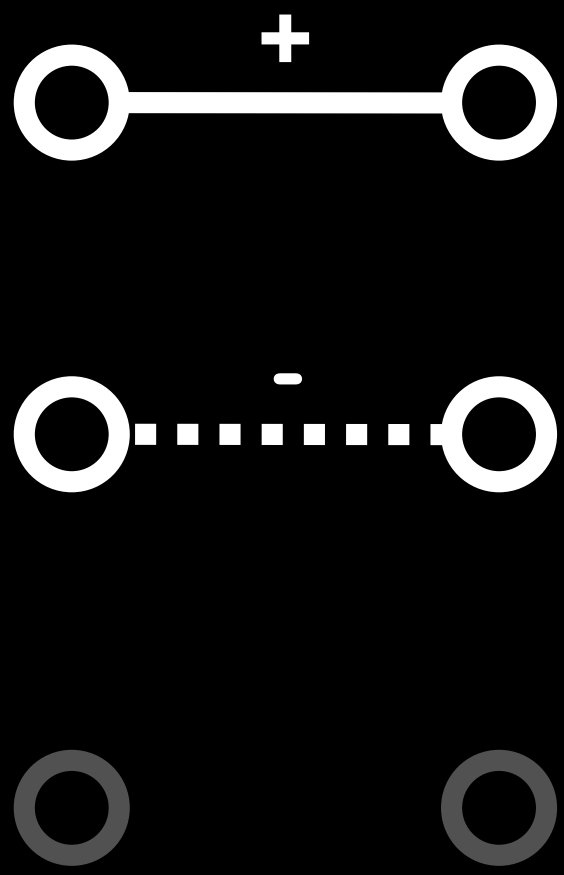
But then v-u is not a bridge



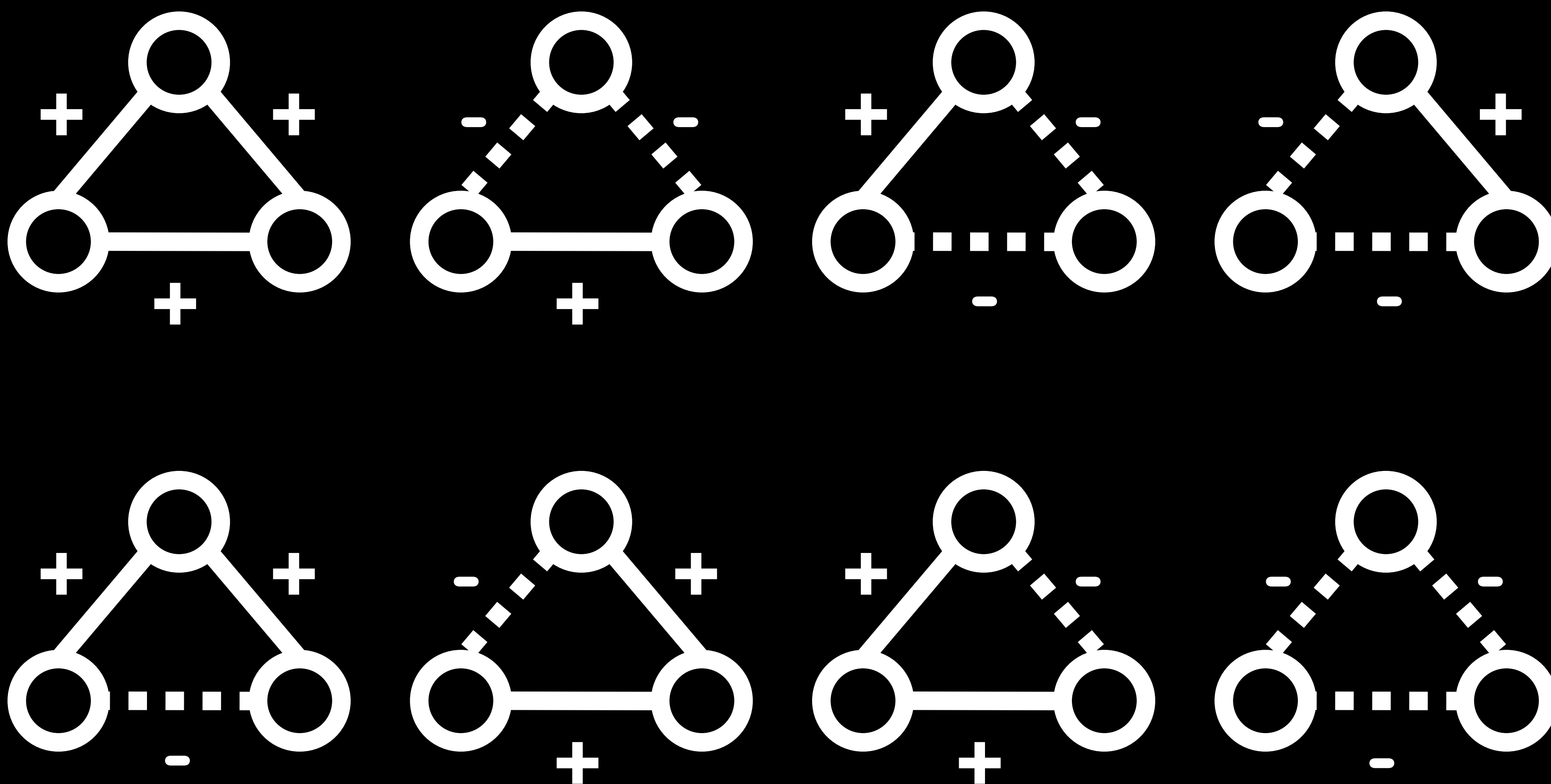
Signed Graphs

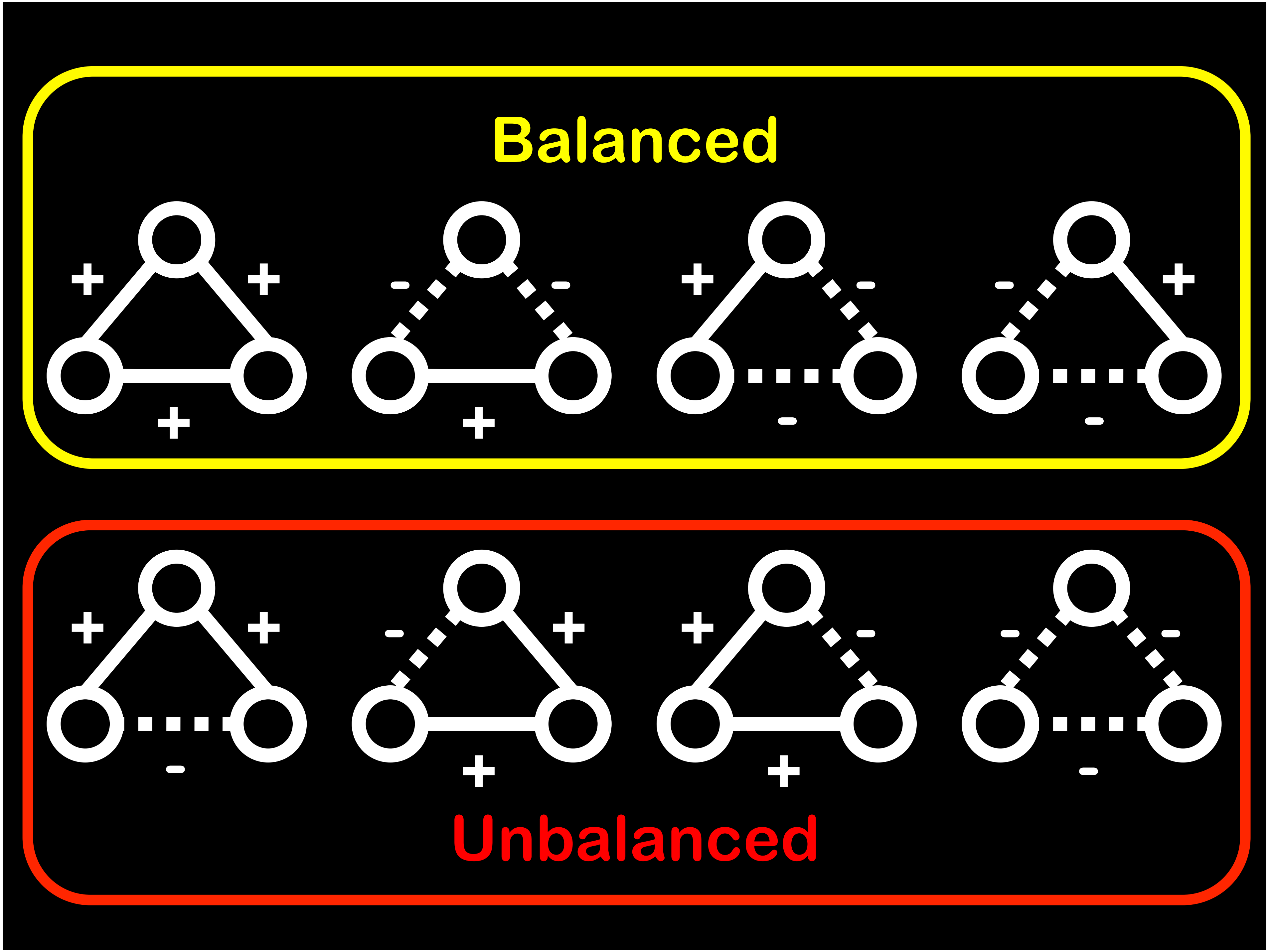


Two-Node Signed Graphs



Three-Node Signed Graphs




Balanced
+
+
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-
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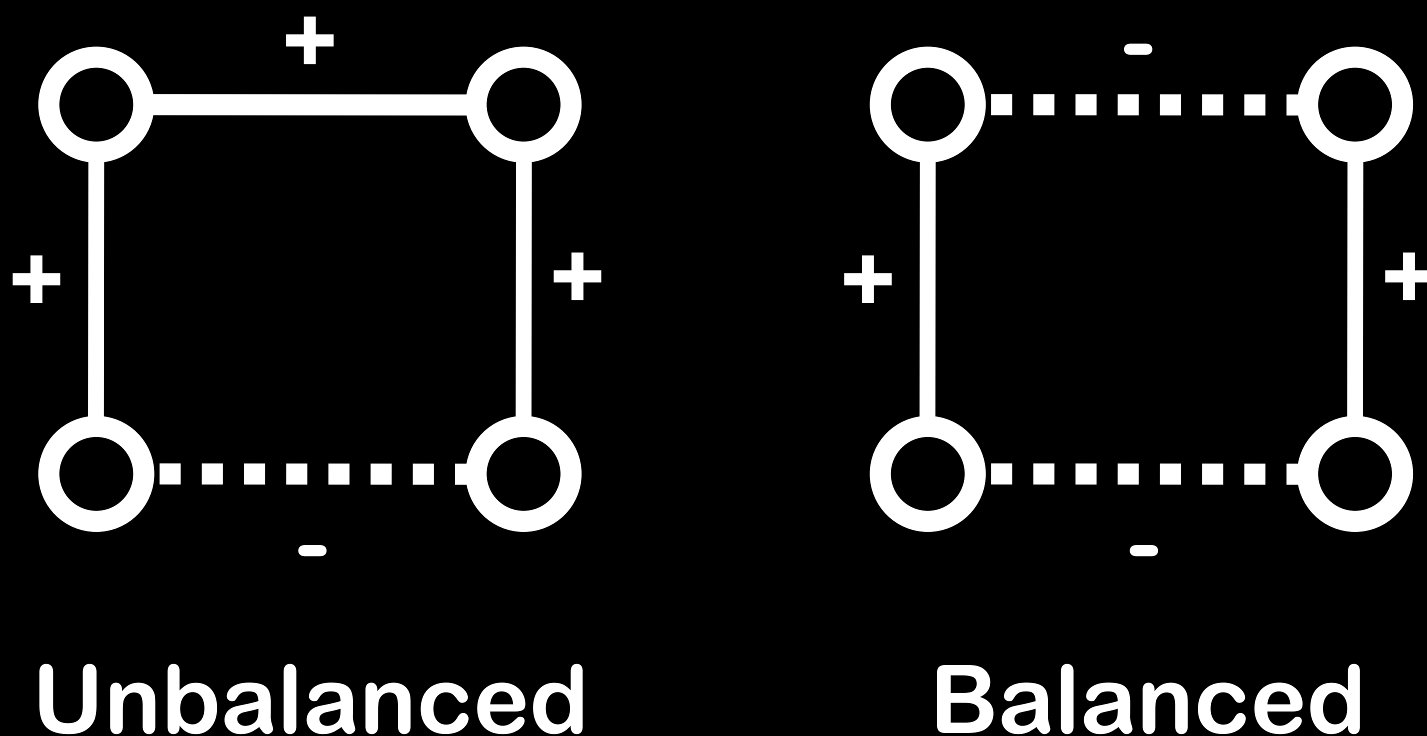
Unbalanced

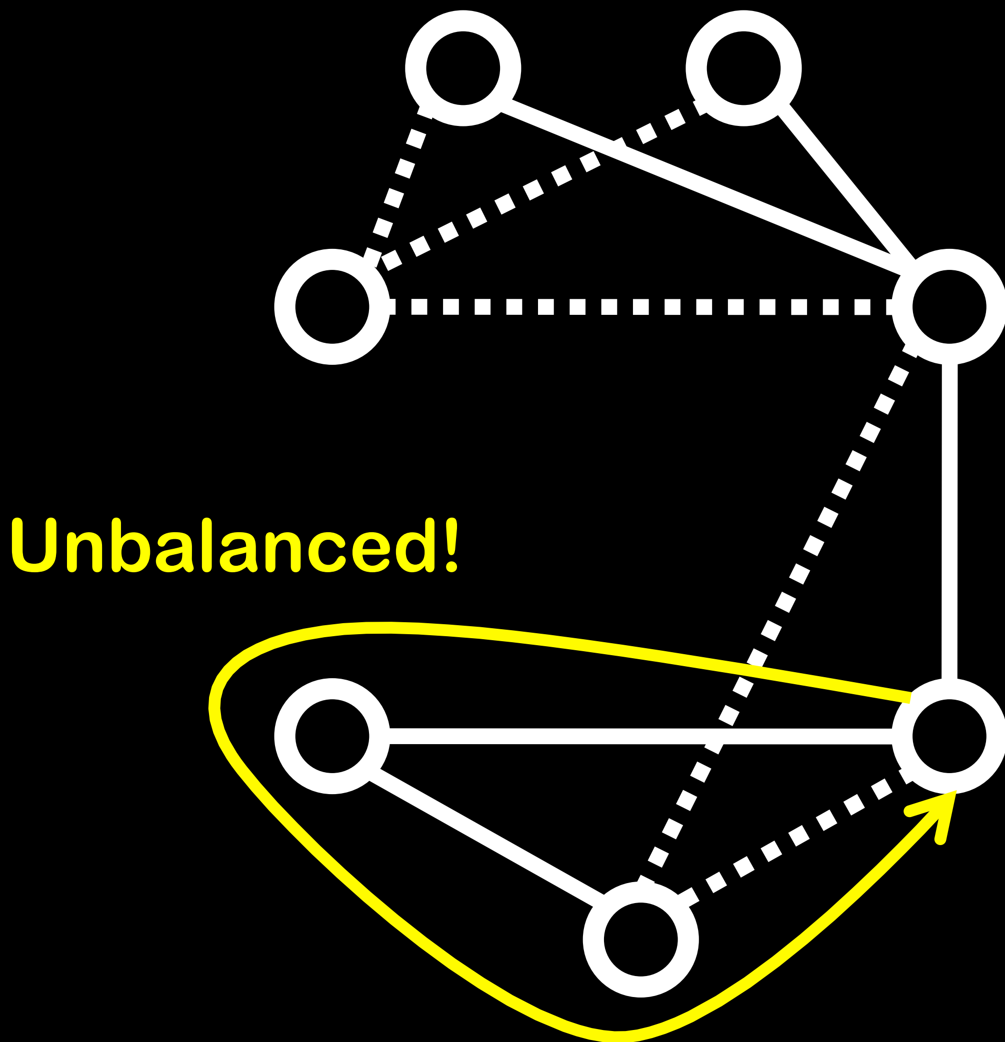
Four Node Cycles

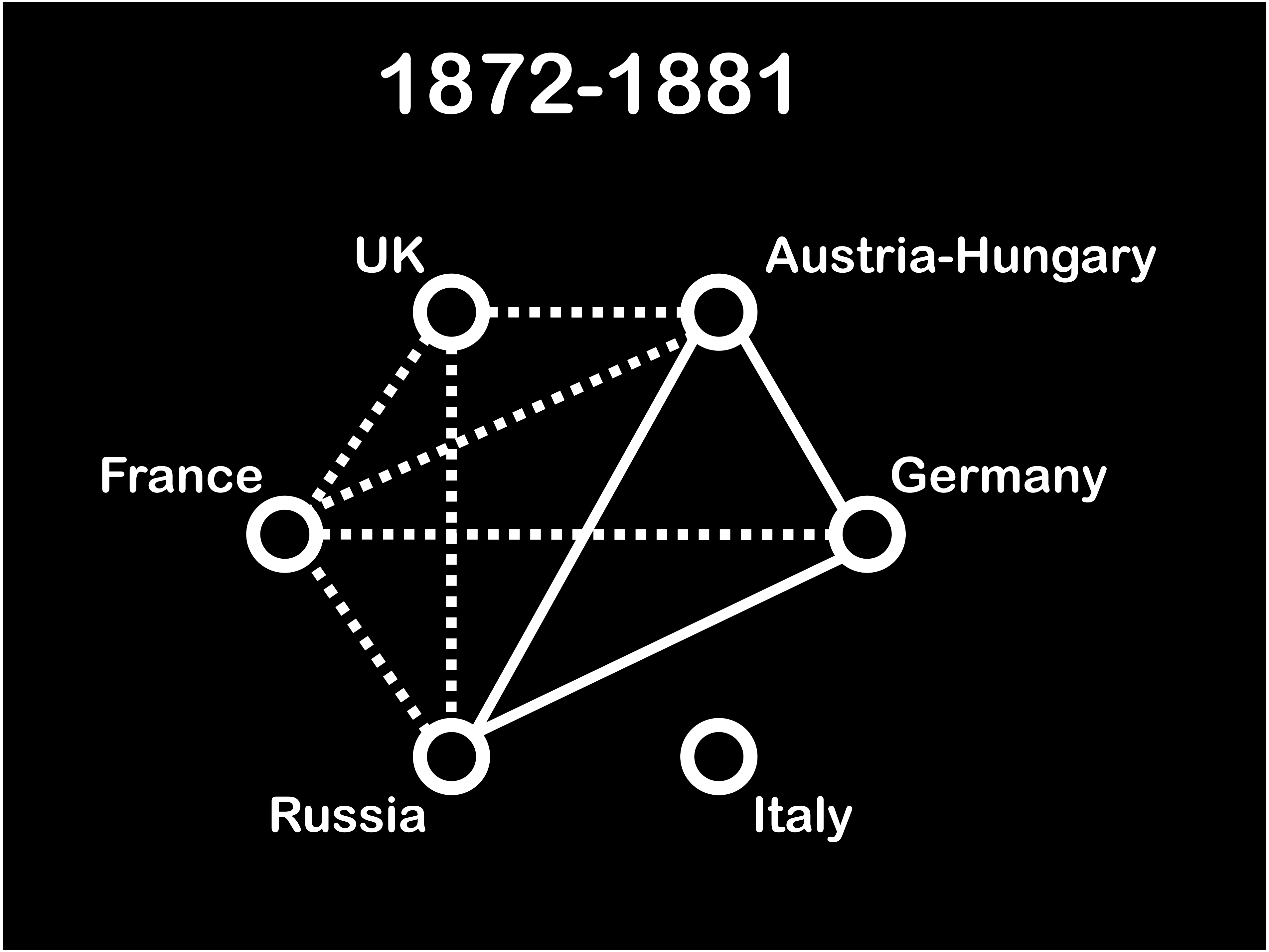


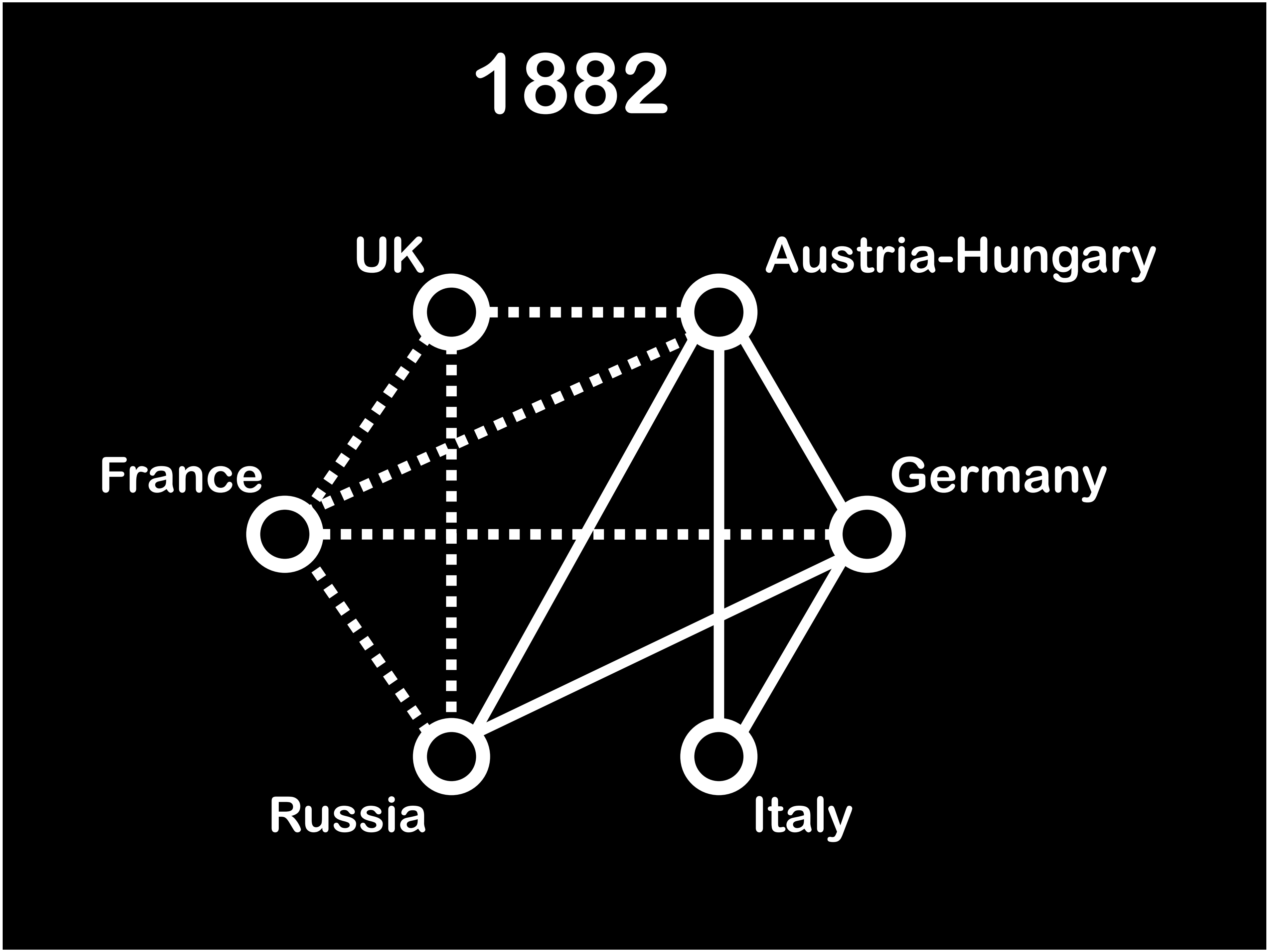
Definition: A cycle is balanced if the product of its signs is positive

Definition: A graph is balanced if all its cycles are balanced

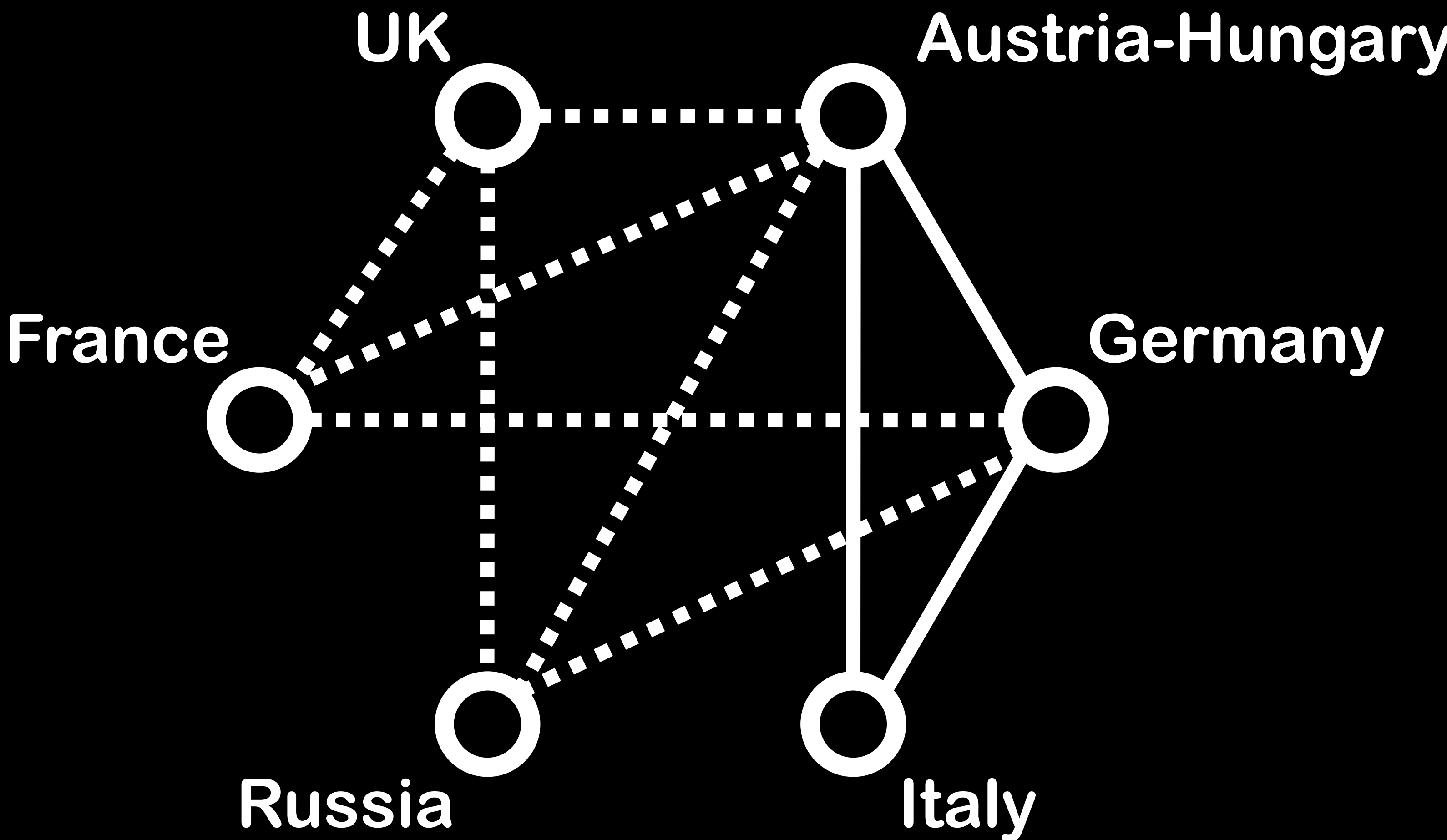
Example

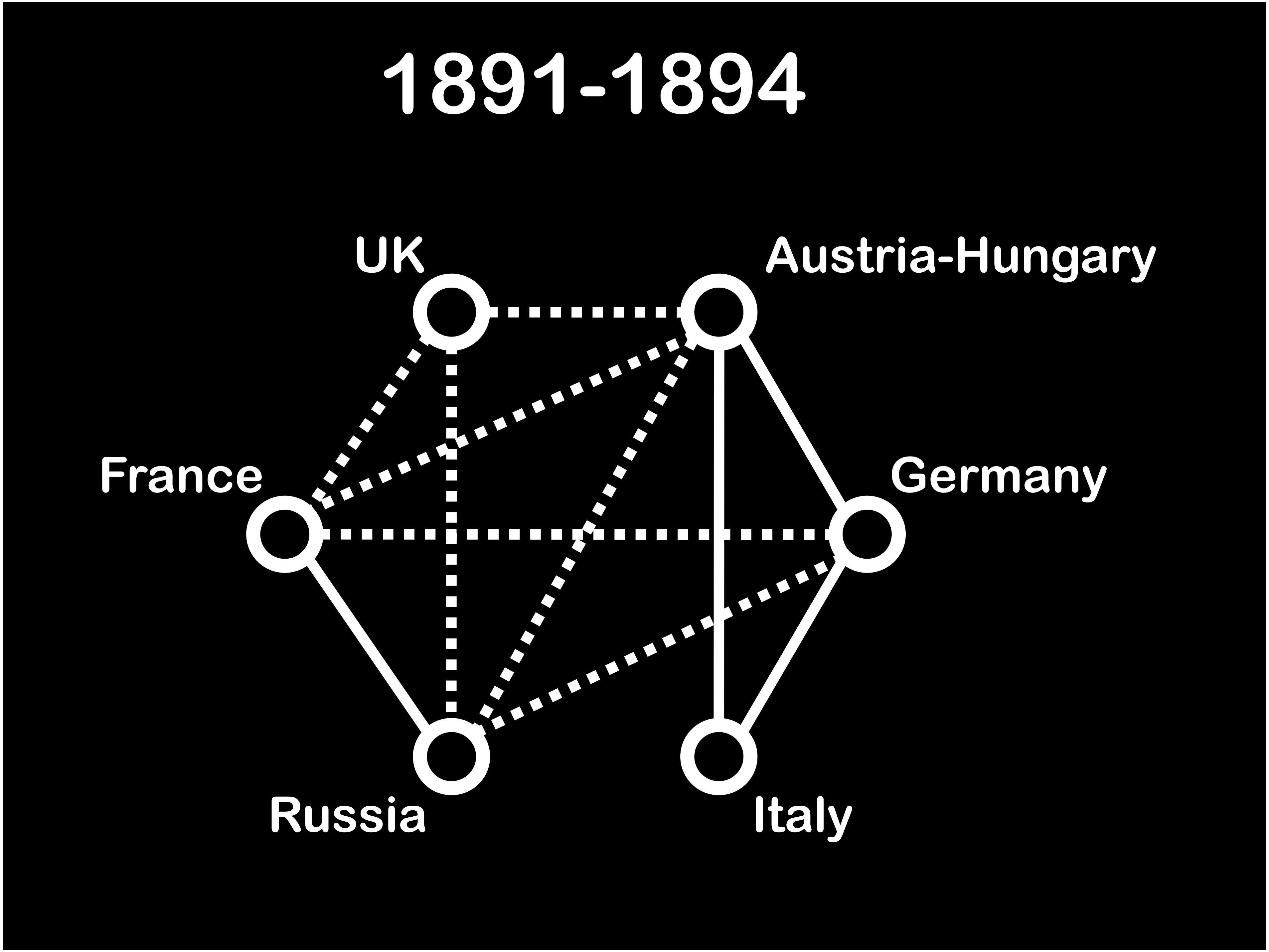


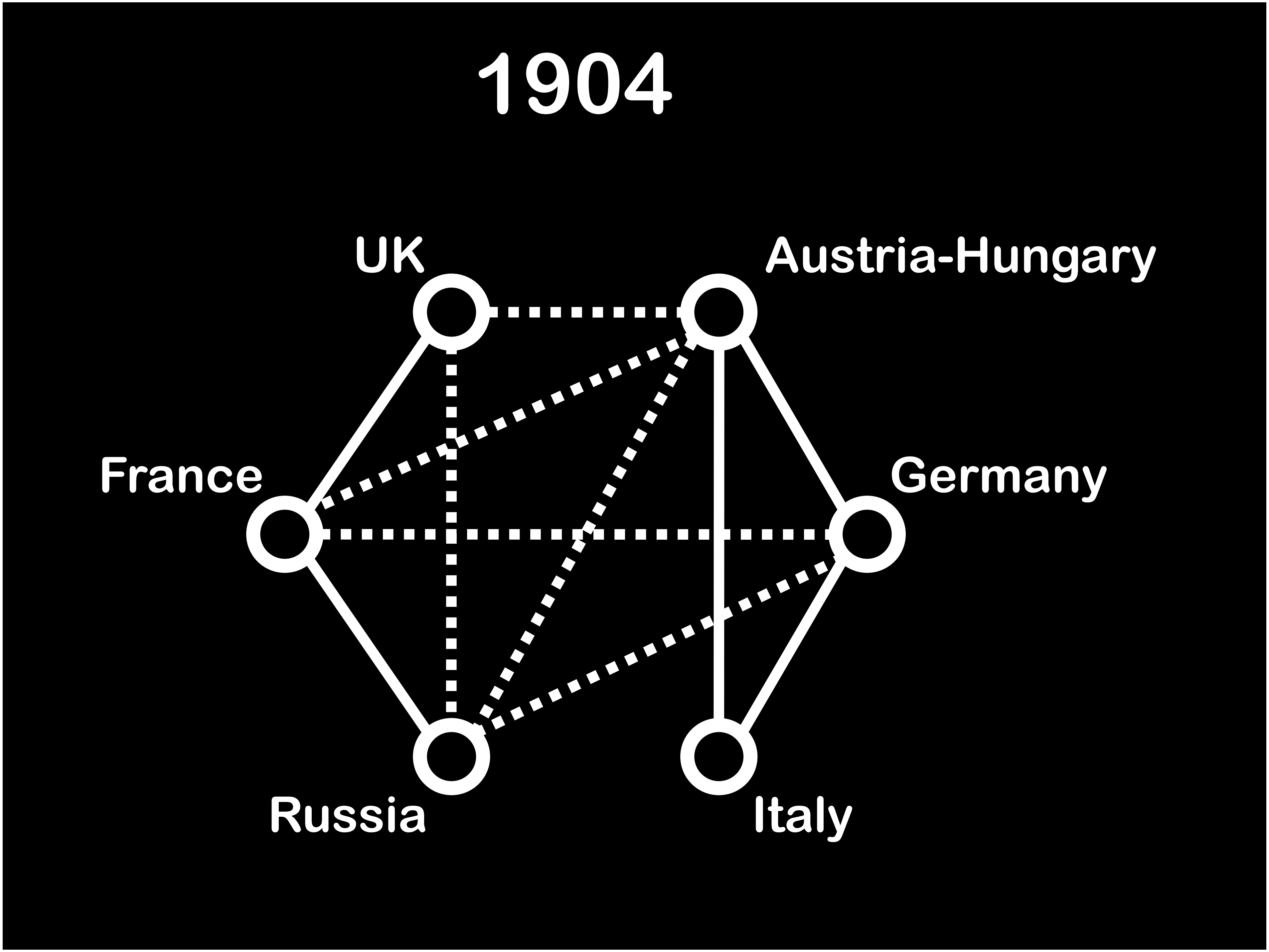

1872-1881
UK
Austria-Hungary
France
Germany
Russia
Italy


1882
UK
Austria-Hungary
France
Germany
Russia
Italy

1890




1891-1894
UK
Austria-Hungary
France
Germany
Russia
Italy


1904
UK
Austria-Hungary
France
Germany
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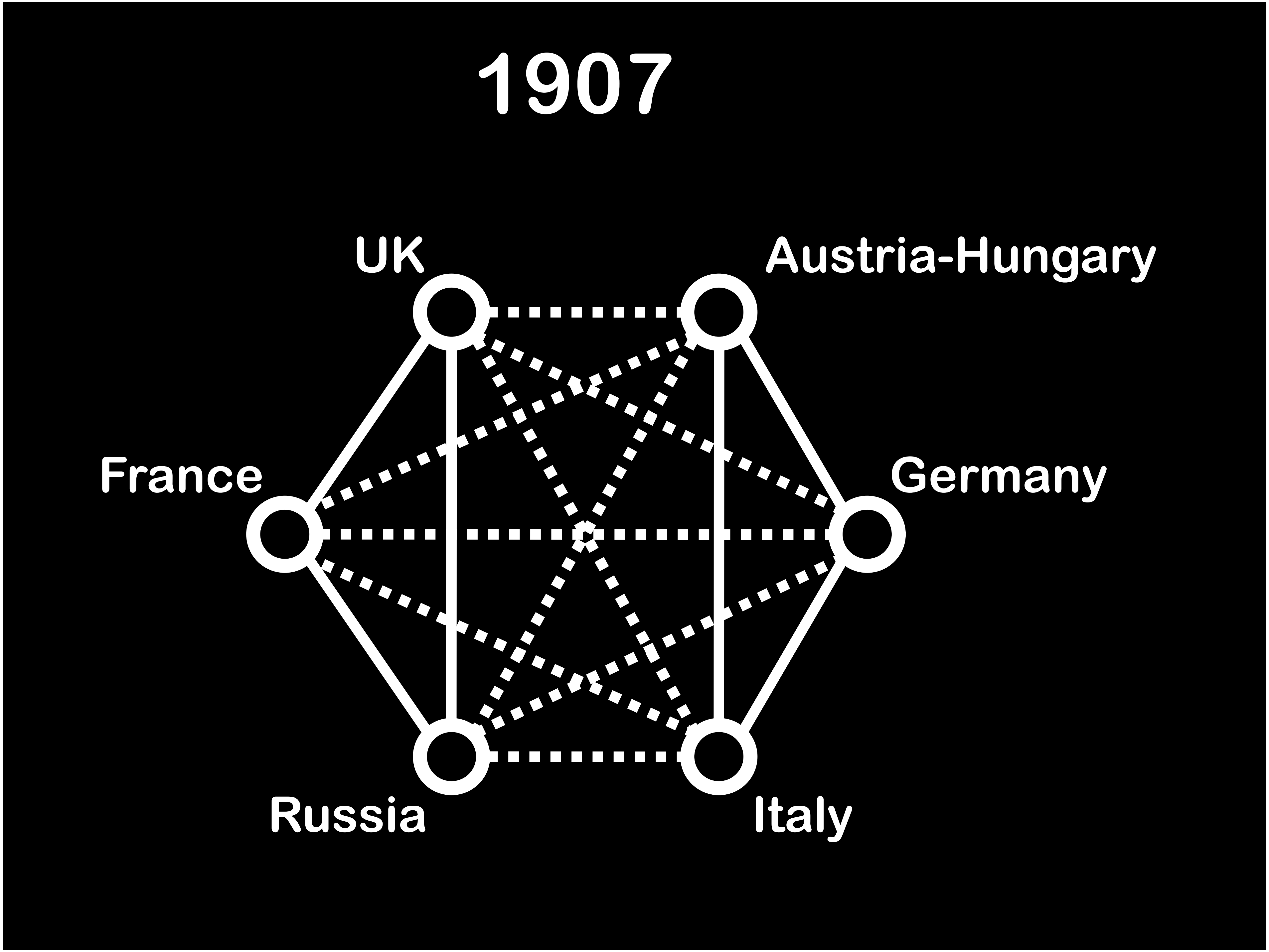
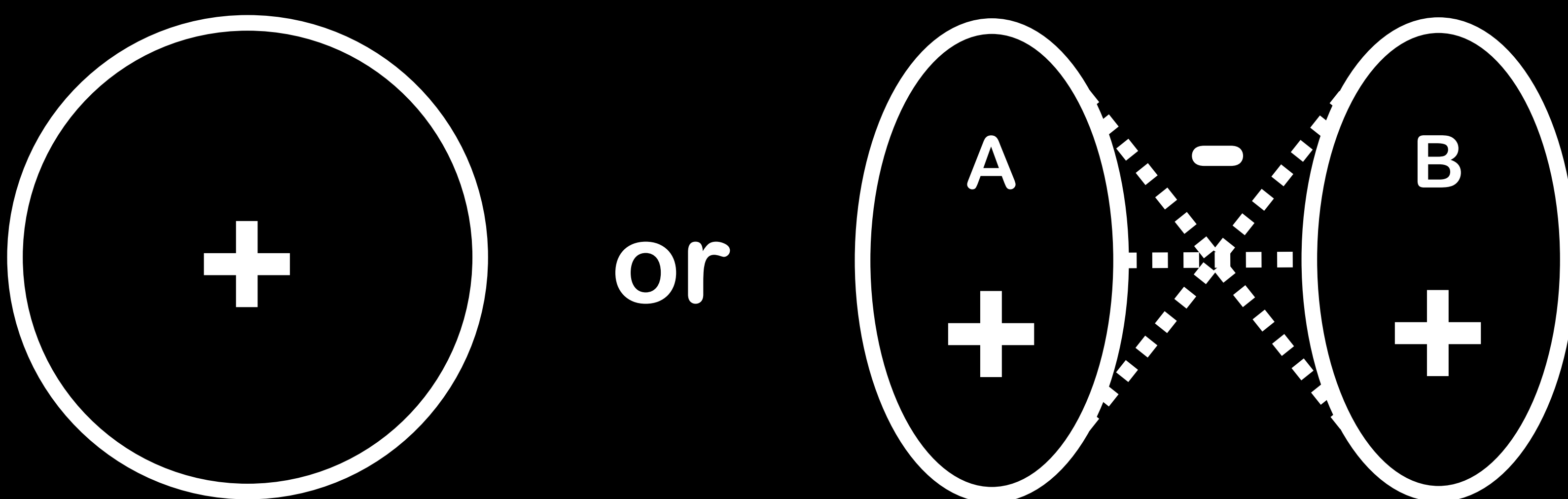
Italy


1907
UK
Austria-Hungary
France
Germany
Russia
Italy

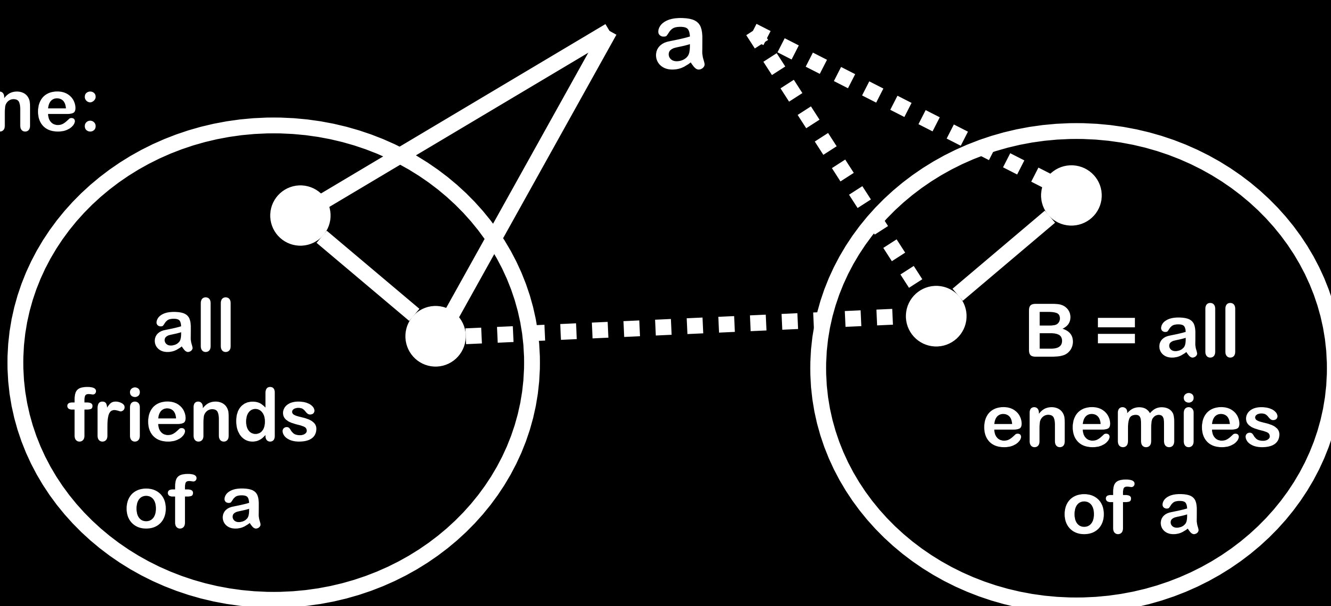
Theorem: If every 3-cycle in a signed complete graph is balanced, then either (1) all nodes are friends, or (2) the nodes can be divided into two groups, A and B, such that every pair of people in A like each other, every pair of people in B like each other, and everyone in A is the enemy of everyone in B.



Proof:

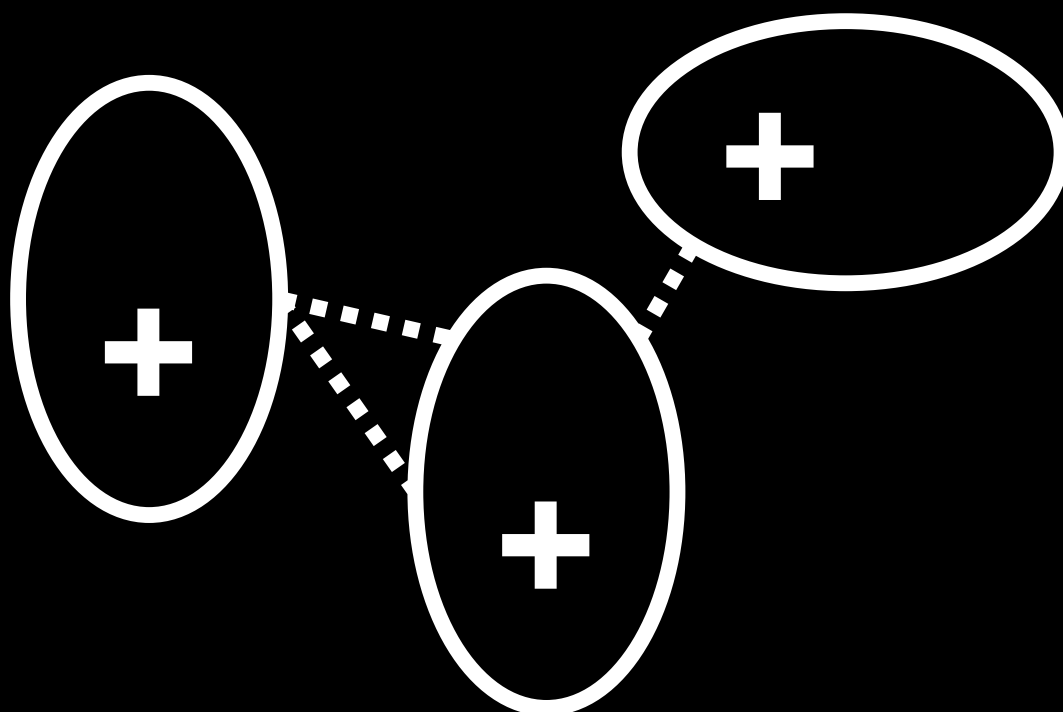
Pack any node:

Define:



1. Every two nodes in A are friends
2. Every two nodes in B are friends
3. Every node in A is an enemy of every node in B

Definition: A signed graph is clusterable if the nodes can be partitioned into a finite number of subsets such every positive edge is between nodes of the same subset, and every negative edge is between nodes of different subsets



Theorem: A signed graph has a clustering if and only if the graph contains no cycles which have exactly one negative edge

g2g
ttyl