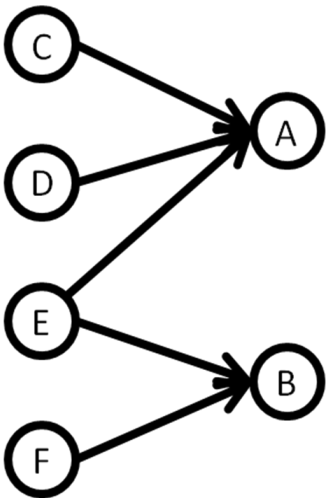
15-396A Science of the Web
Assignment 4
Due November 15, 2011 @ 3:00 pm

Game Theory et al.

1. (50 Points) Let M be a set of advertising slots within a search engine for a given keyword, and let N be a set of bidders for those slots. Let V^M_N denote the maximum total valuation over all possible perfect matchings of slots and bidders – this is simply the sum of all valuations that bidders obtain when the slots are optimally assigned so as to maximize this number. Let M − i denote the set of slots with slots i removed, and let N − j denote the set of bidders with bidder j removed. So if we give slot i to bidder j, then the best total valuation the rest of the bidders could get is V^{M-i}_{N-j}. Let $p_{ij} = V^M_{N-j} - V^{M-i}_{N-j}$. Suppose that slots are assigned as follows:
   A. Ask buyers to announce valuations for the slots. (Note that these announcements need not be truthful.)
   B. Choose a socially optimal assignment of slots to buyers — that is, a perfect matching that maximizes the sum of valuations of each buyer for what they get. This assignment is based on the announced valuations (since that's all we have access to).
   C. Charge each buyer p_{ij}.

   Prove that truthfully announcing valuations is a dominant strategy for each bidder in this type of auction.
2. (30 points) Read up on voting theory in the Kleinberg text and do problems 1 and 2 on page 678 (if you don't have the book for some reason, go to http://www.cs.cornell.edu/home/kleinber/networks-book/networks-book-ch23.pdf The problems are at the end of the chapter on pages 772 – 773).
3. (10 points) Show the values that you get if you run two rounds of computing hub and authority values on the network of Web pages in the figure below. (That is, the values computed by the k-step hub-authority computation when we choose the number of steps k to be 2.) Show the values both before and after the final normalization step, in which we divide each authority score by the sum of all authority scores, and divide each hub score by the sum of all hub scores. (We will call the scores obtained after this dividing-down step the normalized scores. It's fine to write the normalized scores as fractions rather than decimals.)



4. (10 Points) Suppose instead of creating two pages, you create three pages X, Y , and Z, and again try to strategically create links out of them so that X gets ranked as well as possible. Describe a strategy for adding three nodes X, Y , and Z to the network in problem 2, with choices of links out of each, so that when you run the 2-step hub-authority computation (as in problems 2 and 3), and then rank all pages by their authority score, node X shows up in second place.