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## Little Ditties (20 Points Each)

1. Say whether the following claim is true or false, and provide a brief (1-3 sentence) explanation for your answer. *Claim: If player A in a two-person game has a dominant strategy  $s_A$ , then there is a pure strategy Nash equilibrium in which player A plays  $s_A$  and player B plays a best response to  $s_A$ .*
2. Find all Nash equilibria in the game below (pure and randomized). In the payoff matrix below the rows correspond to player A's strategies and the columns correspond to player B's strategies. The first entry in each box is player A's payoff and the second entry is player B's payoff.

	L	R
U	1,1	4,2
D	3,3	2,2

## Seller's Revenue (30 Points)

In this problem we will ask how much a seller can expect to receive for his object in a second-price, sealed-bid auction. Assume that all bidders have independent, private values  $v_i$  which are either 0 or 1. The probability of 0 and 1 are both  $1/2$ .

- a. Suppose there are two bidders. Then there are four possible pairs of their values  $(v_1, v_2)$ : (0, 0), (1, 0), (0, 1), and (1, 1). Each pair of values has probability  $1/4$ . Show that the seller's expected revenue is  $1/4$ . (Assume that if there is a tie at a bid of  $x$  for the highest bid the winner is selected at random from among the highest bidders and the price is  $x$ .)
- b. What is the seller's expected revenue if there are three bidders?
- c. This suggests a conjecture that as the number of bidders increases the seller's expected revenue also increases. In the example we are considering the seller's expected revenue actually converges to 1 as the number of bidders grows. Clearly explain why this should occur.

## Collusion in Auctions (30 Points)

In this question we will consider the effect of collusion between bidders in a second-price, sealed-bid auction. There is one seller who will sell one object using a second-price sealed-bid auction. The bidders have independent, private values drawn from a distribution on  $[0, 1]$ . If a bidder with value  $v$  gets the object at price  $p$ , his payoff is  $v-p$ ; if a bidder does not get the object his payoff is 0. We will consider the possibility of collusion between two bidders who know each others' value for the object. Suppose that

the objective of these two colluding bidders is to choose their two bids as to maximize the sum of their payoffs. The bidders can submit any bids they like as long as the bids are in  $[0, 1]$ .

- a. Let's first consider the case in which there are only two bidders. What two bids should they submit? Explain.
- b. Now suppose that there is a third bidder who is not part of the collusion. Does the existence of this bidder change the optimal bids for the two bidders who are colluding? Explain.

## Bankrupt Us (30 Points)

Listen for details about this question in class. The basic idea is that as a class you need to make us go through our budget on Google AdWords. The class will get points proportional to how much of our budget you go through.

## This is How We Roll (30 Points + X bonus)

Your task is to get someone with at least half a million Twitter followers to link to the class website. The class will get 10 bonus points for each user with more than a million followers that mentions the class. You can individually get extra bonus points if they retweet your mention of the class.